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Fixed-Time Bearing-Based Network Localization

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Abstract
We consider a network localization problem in which a network of multiple nodes needs to estimate their positions based on measured bearing vectors and exchanging several variables. We propose a fixed-time bearing-based estimation law, which guarantees convergence of position estimates in a finite-time independently on the initial estimations. Simulation results are also provided to support the theoretical results.

Keywords: Fixed-time stability, Network localization, Bearing-only measurements, Multi-agent systems.

1. Introduction

In recent years, formation control and network localization problems have attracted a lot of research interests [1, 2]. In a formation control problem, a group of moving autonomous agents needs to achieve a desired formation shape via controlling some geometric variables regarding other agents. As a dual problem to formation control, in a network localization problem, there is a set of stationary sensor nodes, and each node would like to estimate its position based on sensing and exchanging some variables with a few neighbor nodes.

The existing works in the literature mainly focus on designing control/estimation laws which require less sensing and communication resources between the agents/nodes. To this end, the distance-based and the bearing-based approaches are advantageous in comparison with the position-based and displacement-based approaches [3, 4]. While the distance-based approach has been studied extensively for more than a decade [3], the bearing-based approach has just got a considerable attention in recent years since bearing-only control laws can be implemented using only a camera mounted on each agent.

In this paper, we confine our attention on a bearing-based network localization problem. It is worth noting that asymptotic convergence network localization control laws have been proposed in [5]. Furthermore, to enhance the convergence rate, finite-time bearing-only formation control laws have been proposed in [6-8] so that the agents can achieve a target formation after a finite time. Since formation control and network localization are dual problems, it is quite straightforward to apply the finite-time formation control laws in [7] to the network localization problem. However, a disadvantage of finite-time control laws is that the (finite) convergence time depends on the initial condition of the system. As a result, although the target formation in [7, 8] can be achieved in a finite time, it is no guarantee that at a specified time T, the agents are in the desired formation shape or not. This drawback of finite-time controllers can be remedied by the fixed-time control design method in [9]. In simple words, a fixed-time controller guarantees convergence of the system after a finite time T, for all initial conditions. Thus, the main objective of this paper is designing a bearing-only control law that solves the bearing-based network localization problem in a fixed time. The fixed-time analysis is given, and simulation results are also provided.

The remainder of this paper is organized as follows. Section 2 reviews the background on bearing rigidity, bearing-only based network localization, and fixed-time stability condition. In Section 3, we propose the control law and show that it guarantees a fixed-time convergence. Section 4 contains simulation results and Section 5 concludes the paper.

2. Preliminaries

2.1. Fixed-time Stability

Consider the following system

$$\dot{x} = f(t,x),\ x(0) = x_0 \tag{1}$$

where $x \in \mathbb{R}^n$ is the vector of system states, and $f: \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function. If $f(t,x)$ is discontinuous, solutions of (1) are understood in Filippov sense [10]. Assume that the system (1) has zero equilibrium point.

Definition 2.1 ([11]) The equilibrium point $x = 0$ of the system (1) is globally finite-time stable if it is globally asymptotically stable and any solution $x(t,x_0)$ of (1) reaches 0 at some finite time moment, i.e. $x(t,x_0) = 0, \forall t \geq T(x_0)$, where $T: \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ is the so-called settling-time function.

Definition 2.2 ([9]) The equilibrium point $x = 0$ of the system (1) is said to be globally fixed-time stable if it is globally finite-time stable and the settling-time function $T(x_0) \leq T_{\max}$, $\forall x_0 \in \mathbb{R}^n$.

Definition 2.3 ([12]) If there exists a continuous radially unbounded function $V: \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ such that

1. $V(x) = 0 \Leftrightarrow x = 0$;

2. Any solution $x(t)$ of (1) satisfies the inequality $D^*V(x(t)) \le -\alpha V^p(x(t)) - \beta V^q(x(t))$ for some $\alpha, \beta > 0$, $p = 1 - \frac{1}{2\mu}$, $q = 1 + \frac{1}{2\mu}$, $\mu > 1$,

then the origin is globally fixed-time stable for system (1) and the following estimate holds:

$$T_{\max} := \frac{\pi\mu}{\sqrt{\alpha\beta}}, \ \forall x_0 \in \mathbb{R}^n .$$

2.2. Bearing-Based Network Localization

Consider a sensor network of n nodes located at $p_i \in \mathbb{R}^d$ $(d = 2,\ 3)$, $i = 1,...,n$, in the d-dimensional global reference frame. Assume that each node does not know its global position, and thus it has an estimate $\hat{p}_i \in \mathbb{R}^d$. To localize the position, we further assume that each node has its own sensing and communication capabilities. The sensing and communication topologies between the nodes are characterized by a connected, undirected graph $G = (V, E)$, where $V = \{1,...,n\}$ is the node set, $E = \{e_1,...,e_m\} \subset V \times V$ is the edge set [13]. An edge $(i, j) \in E$ implies that two nodes i and j can exchange their position estimates with each other, and they can also sense the directional information (or the bearing vector) with regard to each other. Specifically, if $e_k = (i, j) \in E$, node $i\,(1 \le i \le n)$ can sense the bearing vector

$$g_{ij} = \frac{p_j - p_i}{\left\|p_j - p_i\right\|} = \frac{z_{ij}}{\left\|z_{ij}\right\|} (= \frac{z_k}{\left\|z_k\right\|} = g_k)$$

to nodes j, and node j can sense the bearing vector

$$g_{ji} = \frac{p_i - p_j}{\left\|p_i - p_j\right\|} (= -g_{ij})$$

to node i. Here $z_k = z_{ij}$ is the displacement vector between two nodes i and j. The projection matrix associated with the bearing vector g_{ij} is defined as $P_{g_{ij}} = I_d - g_{ij}g_{ij}^T$. Matrix $P_{g_{ij}}$ projects a vector into the orthogonal complement space of $span\{g_{ij}\}$. It is not hard to verify that $P_{g_{ij}} = P_{g_{ij}}^2 = P_{g_{ij}}^T$, $P_{g_{ij}}$ is positive semidefinite, and $Null(P_{g_{ij}}) = span(g_{ij})$.

Let $N_i = \{j \in V \mid (i, j) \in E\}$ be the neighbor set of node i, then, the locally available information of a node i includes its estimated position $\hat{p}_i$, the set of measured bearing vectors $\{g_{ij}\}_{j \in N_i}$, and the estimated positions $\{\hat{p}_j\}_{j \in N_i}$ received from its neighbor nodes via wireless communication (see Fig. 1). Consider an arbitrarily orientation of edges in G, that is, for each edge $e_k = (i, j)$, we assign i as the starting vertex, j as the

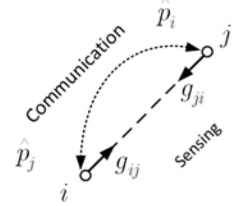

Fig. 1: Nodes i and j measure the bearing vectors g_{ij}, g_{ji} and communicate their estimated positions $\hat{p}_i$, $\hat{p}_j$.

end vertex and the edge is directed from i to j. The incidence matrix $H = [h_{ij}]_{m \times n}$ characterizes the relation between vertices and edges in G corresponding to this orientation is defined as follows:

$$h_{ki} = \begin{cases} -1, & \text{if } e_k = (i, j), \\ 1, & \text{if } e_k = (j, i), \\ 0, & \text{otherwise.} \end{cases}$$

If the graph G is connected, we always have $Null(H) = span(1_n)$. Let $z = [z_1^T,...,z_m^T]^T$, then there holds $z = (H \otimes I_d)p = \bar{H}p$, where '$\otimes$' denotes the Kronecker product.

A network, denoted by (G, p), is described by a graph G and a configuration $p = [p_1^T,...,p_1^T]^T$ of G in the space. The rigidity matrix of (G, p) is defined as

$$R_B(p) = \begin{bmatrix} P_{g_1} & & \\ & \ddots & \\ & & P_{g_m} \end{bmatrix} (H \otimes I_d) = diag(P_{g_k})\bar{H}. \quad (2)$$

where $diag(P_{g_k})$ is the block diagonal matrix of m projection matrices $P_{g_1},...,P_{g_m}$. For any bearing rigidity matrix, we have $Null(R_B(p)) \supseteq Range([1_n \otimes I_d,\ p])$.

This paper aims to design a position estimation law for each node based on only local information so that they can determine a configuration $\hat{p}^* = [\hat{p}_1^{*T},...,\hat{p}_n^{*T}]^T$ satisfying all measured bearing vectors between the agents in a fixed time ($\hat{p}^*$ is different from p by only a translation and a scaling). In other words, we would like to design update laws for $\hat{p}_i(t)$ so that

$$\hat{g}_{ij}(t) = \frac{\hat{p}_j(t) - \hat{p}_i(t)}{\left\|\hat{p}_j(t) - \hat{p}_i(t)\right\|} = g_{ij}, \ \ i, j \in V,\ \forall i \ne j, \quad (3)$$

$\forall t \ge T_{\max} > 0$, and $T_{\max}$ is independent on the initial estimations $\hat{p}(0)$. From now on, we will refer to this problem as the fixed- time bearing- based network localization problem.

In order to solve the problem, the following assumption on the network (G, p) will be adopted:

Assumption 2.4 The network (G, p) is infinitesimally bearing rigid in $\mathbb{R}^d$, or i.e., we have $Null(R_B) = Range([1_n \otimes I_d,\ p])$.

In this paper, we would like to keep the background on bearing rigidity theory as minimal as possible. For further results on bearing rigidity theory, the readers are referred to [14].

3. Fixed-Time Bearing-Based Network Localization

The proposed control law

For a vector $v = [v_1, \dots, v_d]^T \in \mathbb{R}^d$, we will denote

$$sig(v)^\alpha = [\mathrm{sgn}(v_1)\,|\,v_1\,|^\alpha, \dots, \mathrm{sgn}(v_d)\,|\,v_d\,|^\alpha]^T$$

and $\quad |\,v\,|^\alpha = [|\,v_1\,|^\alpha, \dots, |\,v_d\,|^\alpha]^T$.

The following position estimation law is proposed for each node $i = 1, \dots, n$:

$$\dot{\hat{p}}_i = \sum\nolimits_{j \in N_i} P_{\hat{g}_{ij}} \left(sig(P_{\hat{g}_{ij}} g_{ij})^\alpha + sig(P_{g_{ij}} g_{ij})^\beta \right), \qquad (4)$$

where $\alpha \in (0,1)$ and $\beta = 2 - \alpha > 1$ are two control parameters. Note that in Eqn. (4), $P_{\hat{g}_{ij}}$ can be calculated from $\hat{p}_i$ and the communicated variables $\hat{p}_j$, and g_{ij} are measured by agent i. We can rewrite the estimation law (4) in matrix form as follows:

$$\dot{\hat{p}} = \bar{H}^T diag(P_{\hat{g}_k})(sig(diag(P_{\hat{g}_k})g)^\alpha + sig(diag(P_{\hat{g}_k})g)^\beta), \quad (5)$$

where $g = [g_1^T, \dots, g_m^T]^T$, and $\hat{g} = [\hat{g}_1^T, \dots, \hat{g}_m^T]^T$. Let $\bar{\hat{p}} = \frac{1}{n}\sum_{i=1}^n \hat{p}_i = \frac{1}{n}(1_n \otimes I_d)^T \hat{p}$, $\hat{r} = \hat{p} - 1 \otimes \bar{\hat{p}}$, and $\hat{s} = \|\hat{r}\|$ be the estimated centroid and estimated scale, respectively, we have the following lemma:

Lemma 3.1 The estimated centroid is invariant while the scale under the estimation law (5).

Proof. The result follows from

$$\dot{\bar{\hat{p}}}(t) = \frac{1}{n}(1_n \otimes I_d)^T \dot{\hat{p}} = \frac{1}{n}(1_n \otimes I_d)^T \bar{H}^T diag(P_{\hat{g}_k})(\bullet)$$
$$= \frac{1}{n}(H1_n \otimes I_d)^T diag(P_{\hat{g}_k})(\bullet) = 0,$$
$$\dot{\hat{s}}(t) = \frac{(\hat{p} - 1 \otimes \bar{\hat{p}})^T \bar{H}^T diag(P_{\hat{g}_k})(\bullet)}{\|\hat{p} - 1 \otimes \bar{\hat{p}}\|}$$
$$= \frac{\hat{p}^T R_B^T(\hat{p})(\bullet)}{\|\hat{p} - 1 \otimes \bar{\hat{p}}\|} - \frac{(H1 \otimes \bar{\hat{p}})^T diag(P_{\hat{g}_k})(\bullet)}{\|\hat{p} - 1 \otimes \bar{\hat{p}}\|} = 0,$$

where '$\bullet$' is $(sig(diag(P_{\hat{g}_k})g)^\alpha + sig(diag(P_{\hat{g}_k})g)^\beta)$. ■

From the invariance of the estimated centroid and scale, we can then have the following lemma regarding the equilibrium points of (5).

Lemma 3.2 The system (5) has two isolated equilibria: $\hat{p} = \hat{p}^*$ corresponding to $\hat{g}_k = g_k, \forall k = 1, \dots, m$, and $\hat{p} = \hat{p}'$ corresponding to $\hat{g}_k = -g_k, \forall k = 1, \dots, m$.

Proof. From equation $\dot{\hat{p}}(t) = 0$, it follows that

$$\hat{p}^{*T}\dot{\hat{p}} = \hat{p}^{*T}\bar{H}^T diag(P_{\hat{g}_k})(sig(diag(P_{\hat{g}_k})g)^\alpha + sig(diag(P_{\hat{g}_k})g)^\beta)$$
$$= \sum_{k=1}^m \hat{z}_k^{*T} P_{\hat{g}_k}(sig(P_{\hat{g}_k} g_k)^\alpha + sig(P_{\hat{g}_k} g_k)^\beta)$$
$$= \sum_{k=1}^m \|\hat{z}_k^*\| g_k^T P_{\hat{g}_k}(sig(P_{\hat{g}_k} g_k)^\alpha + sig(P_{\hat{g}_k} g_k)^\beta) = 0,$$

which implies that $\hat{g}_k = \pm g_k, \forall k = 1, \dots, m$. The remaining proof follows from Assumption 2.4 and a similar reasoning as in [Thm. 10, 14]. ■

Let $\delta = [\delta_1^T, \dots, \delta_n^T]^T = \hat{p} - \hat{p}^*$, and $\hat{r}^* = \hat{p}^* - 1 \otimes \bar{\hat{p}}$, since the centroid is invariant, it follows that $\delta = \hat{r} - \hat{r}^*$.

Thus, we can rewrite (5) as

$$\dot{\delta} = \bar{H}^T diag(P_{\hat{g}_k})(sig(diag(P_{\hat{g}_k})g)^\alpha + sig(diag(P_{\hat{g}_k})g)^\beta), \quad (6)$$

Moreover, as the scale is invariant, it follows that δ evolves on the sphere $\|\delta + \hat{r}^*\| = \|\hat{r}\| = \|\hat{r}^*\|$ and (6) has two equilibria in this sphere: $\delta = 0$ and $\delta = -2\hat{r}^*$. Next, we examine the stability of these equilibria. In the analysis, we will always assume that $\hat{p}_i(t) \neq \hat{p}_j(t), \forall t \geq 0$ so that the vectors $\hat{g}_{ij}$ are always defined.

Theorem 3.3 The equilibrium $\delta = 0$ of (6) is asymptotically stable.

Proof. Consider the Lyapunov function $V = \frac{1}{2}\|\delta\|^2$, which is positive definite, radially unbounded, and continuously differentiable. We have

$$\dot{V} = \delta^T \dot{\delta}$$
$$= (\hat{p} - \hat{p}^*)^T \bar{H}^T diag(P_{\hat{g}_k})(sig(diag(P_{\hat{g}_k})g)^\alpha + sig(diag(P_{\hat{g}_k})g)^\beta)$$
$$= -\hat{p}^{*T}\bar{H}^T diag(P_{\hat{g}_k})(sig(diag(P_{\hat{g}_k})g)^\alpha + sig(diag(P_{\hat{g}_k})g)^\beta)$$
$$= -\sum_{k=1}^m \|\hat{z}_k^*\| g_k^T P_{\hat{g}_k}(sig(P_{\hat{g}_k} g_k)^\alpha + sig(P_{\hat{g}_k} g_k)^\beta)$$
$$= -\sum_{k=1}^m \|\hat{z}_k^*\| \sum_{i=1}^d (|\,[P_{\hat{g}_k} g_k]_i\,|^{\alpha+1} + |\,[P_{\hat{g}_k} g_k]_i\,|^{\beta+1}) \leq 0,$$

and the inequality holds if and only if $\hat{g}_k = \pm g_k, \forall k = 1, \dots, m$, or i.e., $\delta = 0$ or $\delta = -2\hat{r}^*$. By LaSalle's invariance principle, $\delta \to \{0, -2\hat{r}^*\}$. Since two equilibria are isolated, consider a neighborhood of $\delta = 0$ which does not contains $\delta = -2\hat{r}^*$, then $\dot{V} < 0$, for all $\delta \neq 0$ in that neighborhood. Thus, $\delta = 0$ is locally asymptotically stable. ■

For the equilibrium $\delta = -2\hat{r}^*$, we consider the function $V = \frac{1}{2}\|\delta + 2\hat{r}^*\|^2$ and follow a similar proof as in Theorem 3.3, it can be proved that $\dot{V} \geq 0$, and thus $\delta = -2\hat{r}^*$ is unstable. We thus conclude that $\delta \to 0$ asymptotically if $\delta(0) \neq -2\hat{r}^*$.

Before showing fixed-time stability of the equilibrium $\delta = 0$, we state the following lemma, whose proof is similar to [14] and will be omitted.

Lemma 3.4 Under the estimation law (6), the following inequality holds

$$\left\|\hat{z}_k\right\| \le 2\hat{s}\sqrt{n-1},\ k=1,\ldots,m. \qquad (7)$$

We can now prove the main theorem of this paper.

Theorem 3.5 Suppose that $\delta(0) \neq -2\hat{r}^*$, the equilibrium $\delta = 0$ of (6) is locally fixed- time stable.

Proof. Consider an arbitrarily closed neighborhood *D* of $\delta = 0$ which does not contain $-2\hat{r}^*$. Let $\varepsilon = \min_{k=1,\ldots,m}\left\|\hat{z}^*(t)\right\|$, we can write

$$\dot{V} \le -\varepsilon\sum_{k=1}^{m}\sum_{i=1}^{d}(|[P_{\hat{g}_k}g_k]_i|^{\alpha+1} + |[P_{\hat{g}_k}g_k]_i|^{\beta+1}).$$

For $0<r<l$, and $x \in \mathbb{R}^{dm}$, we have the following norm inequality $\left\|x\right\|_l \le \left\|x\right\|_r \le (dm)^{\left(\frac{1}{r}-\frac{1}{l}\right)}\left\|x\right\|_l$. Thus, for $1<\alpha+1<2$ we have

$$\sum_{k=1}^{m}\sum_{i=1}^{d}|[P_{\hat{g}_k}g_k]_i|^{\alpha+1} \ge \left(\sum_{k=1}^{m}\sum_{i=1}^{d}|[P_{\hat{g}_k}g_k]_i|^2\right)^{\frac{\alpha+1}{2}}, \qquad (8)$$

and for $2<\beta+1$, there holds

$$\sum_{k=1}^{m}\sum_{i=1}^{d}|[P_{\hat{g}_k}g_k]_i|^{\beta+1} \ge (dm)^{1-\beta}\left(\sum_{k=1}^{m}\sum_{i=1}^{d}|[P_{\hat{g}_k}g_k]_i|^2\right)^{\frac{\beta+1}{2}} \qquad (9)$$

Moreover, following a similar proof as in [Thm. 5, 8], we have

$$\begin{aligned}\sum_{k=1}^{m}\sum_{i=1}^{d}|[P_{\hat{g}_k}g_k]_i|^2 &= \sum_{k=1}^{m}g_k^TP_{\hat{g}_k}g_k \\ &= \sum_{k=1}^{m}\hat{g}_k^TP_{g_k}\hat{g}_k = \sum_{k=1}^{m}\frac{1}{\left\|\hat{z}_k\right\|^2}\hat{z}_k^TP_{g_k}\hat{z}_k \\ &\ge \frac{1}{(2\hat{s}\sqrt{n-1})^2}\left(\sum_{k=1}^{m}(\hat{z}_k-z_k)P_{g_k}(\hat{z}_k-z_k)\right) \\ &\ge \frac{1}{(2\hat{s}\sqrt{n-1})^2}\left(\delta^T\bar{H}^Tdiag(P_{g_k})\bar{H}\delta\right) \\ &\ge \chi\left\|\delta\right\|^2 = 2\chi V,\end{aligned}$$

where $\chi = \frac{\lambda_{d+2}^2(R_B)\phi_0}{(2\hat{s}\sqrt{n-1})^2} > 0$, $\lambda_{d+2}(R_B)$ is the smallest positive eigenvalue of $R_B(p)$, and $\phi_0 > 0$ is a constant that only depends on *D*. The last derivation step is similar to [Thm. 5, 8] and has been omitted for brevity. Thus, combining this with (8) and (9), it follows that

$$\dot{V} \le -\varepsilon(2\chi)^{\frac{\alpha+1}{2}}V^{\frac{\alpha+1}{2}} - \varepsilon(dm)^{1-\beta}(2\chi)^{\frac{\beta+1}{2}}V^{\frac{\beta+1}{2}}. \qquad (10)$$

Let $p = \frac{\alpha+1}{2}$, $q = \frac{\beta+1}{2}$, $k_1 = \varepsilon(2\chi)^{\frac{\alpha+1}{2}}$, and $k_2 = \varepsilon(dm)^{1-\beta}(2\chi)^{\frac{\beta+1}{2}}$, then based on Lemma 2.1, $\delta = 0$ is a fixed-time stable equilibrium of (6), i.e., we have $\delta(t) = 0,\ \forall t \ge T_{\max}$, where

$$T_{\max} = \frac{\pi(\beta-\alpha)}{2\sqrt{k_1k_2}} = \frac{\pi(\beta-\alpha)}{2\varepsilon(2\chi)^{\frac{\alpha+\beta+2}{4}}(dm)^{\frac{1-\beta}{2}}}. \qquad \blacksquare$$

4. Simulation Results

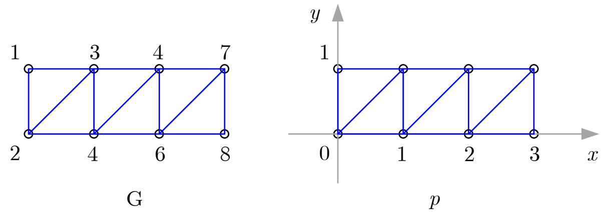

Fig. 2: The graph G and the true configuration p of 10 agents in $\mathbb{R}^2$.

In this section, we consider a sensor network consisting of 10 nodes in the two-dimensional space. The nodes' true positions and the graph describing the sensing and communication topologies between them are shown in Fig. 2. The nodes have initial estimations $\hat{p}(0)$ which is graphically given in Fig. 3. They update their estimations under the estimation law (4) with $\alpha = 0.6,\ \beta = 1.4$.

Simulation results are shown in Fig. 3. It is observed that under the estimation law (4), $\hat{p}(t)$ converges to a final configuration $\hat{p}^*$ differing from the true configuration by translations and a scaling after about 20s. To compare, we also conduct a simulation of the same nodes under the unadjusted estimation law [14]:

$$\dot{\hat{p}}_i = -\sum_{j\in N_i}P_{\hat{g}_{ij}}g_{ij}. \qquad (11)$$

Simulation result in Fig. 4 shows that after 25s, the estimated configuration $\hat{p}(t)$ still does not converge to $\hat{p}^*$.

5. Conclusions

In this paper, we proposed a fixed-time network localization using only bearing measurements. Mathematical analysis and simulations are provided to show the fixed-time convergence of the position estimates under the proposed estimation law. Fixed-time convergence is a powerful property that has not been applied much in multi-agent systems. In future work, we would like to design fixed-time controllers in other problems in multi-agent systems.

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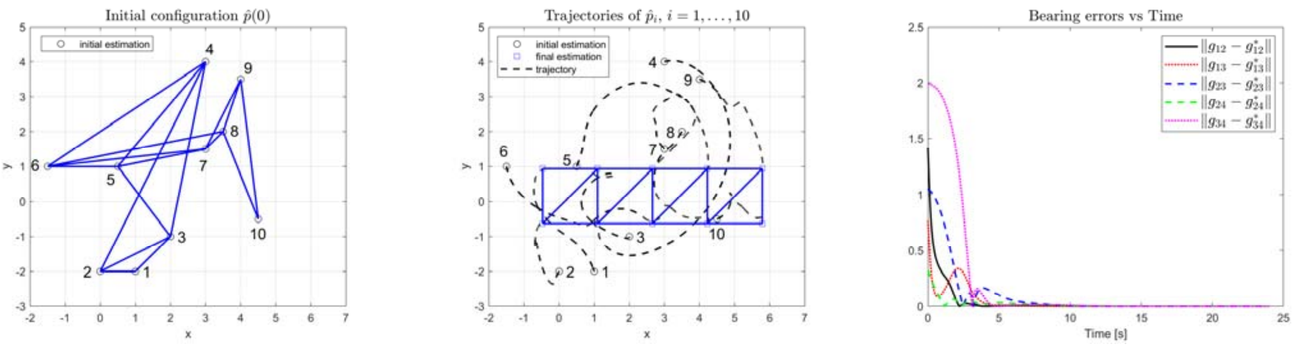

Fig. 3: Initial estimations, trajectories of estimated positions, and bearing errors under estimation law (4).

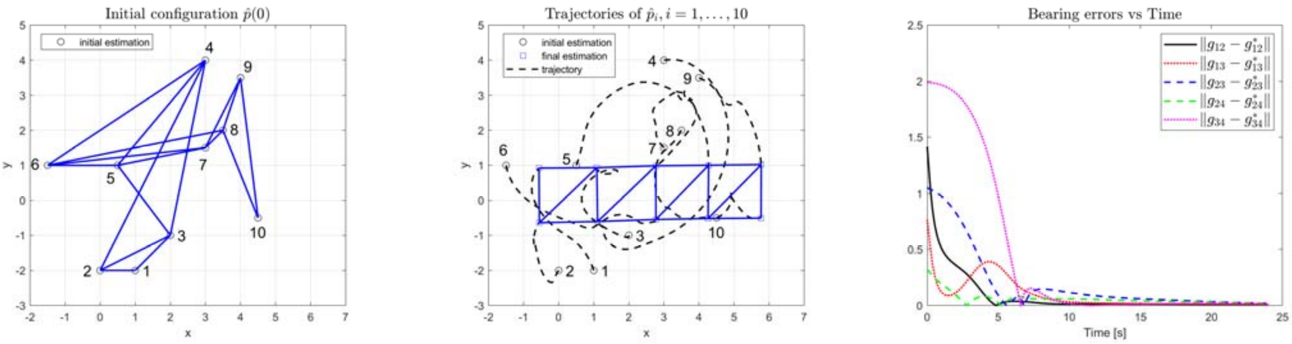

Fig. 4: Initial estimations, trajectories of estimated positions, and bearing errors under the estimation law (11).

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